Please check the examination details below before entering your candidate information

Candidate surname	Other names

Centre Number

Candidate Number

Pearson Edexcel International Advanced Level

Wednesday 18 October 2023

Morning (Time: 1 hour 30 minutes)

Paper reference **WMA13/01**

Mathematics

International Advanced Level
Pure Mathematics P3

You must have:
Mathematical Formulae and Statistical Tables (Yellow), calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 10 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over

P 7 4 3 2 8 A 0 1 3 2

Pearson

1. A curve has equation $y = f(x)$ where

$$f(x) = x^2 - 5x + e^x \qquad x \in \mathbb{R}$$

(a) Show that the equation $f(x) = 0$ has a root, α, in the interval $[1, 2]$

(2)

The iterative formula

$$x_{n+1} = \sqrt{5x_n - e^{x_n}}$$

with $x_1 = 1$ is used to find an approximate value for the root α.

(b) (i) Find the value of x_2 to 4 decimal places.

(ii) Find, by repeated iteration, the value of α, giving your answer to 4 decimal places.

(3)

1. a) $f(x) = x^2 - 5x + e^x = 0$

$f(1) = (1)^2 - 5(1) + e^{(1)} = -1.282$

$f(2) = (2)^2 - 5(2) + e^{(2)} = 1.389$

given that $f(x)$ is continuous between 1 & 2 because there's a sign change

→ root lies between 1 & 2

b) (i) $x_{n+1} = \sqrt{5x_n - e^{x_n}}$

$x_1 = 1$

$x_2 = x_{1+1} = \sqrt{5(1) - e^1} = 1.5105$

(ii) $x_3 = 1.7388$ $x_6 = 1.7340$

$x_4 = 1.7331$ $x_7 = 1.7340$

$x_5 = 1.7342$

consistent to 4 d.p.

$\alpha = 1.7340$

DO NOT WRITE IN THIS AREA

2. The function f is defined by

$$f(x) = \frac{x+3}{x-4} \qquad x \in \mathbb{R}, x \neq 4$$

(a) Find ff(6)
(2)

(b) Find f^{-1}
(3)

The function g is defined by

$$g(x) = x^2 + 5 \qquad x \in \mathbb{R}, x > 0$$

(c) Find the exact value of a for which

$$gf(a) = 7$$

(3)

2. a) $f(x) = \frac{x+3}{x-4}$

$f(6) = \frac{6+3}{6-4} = \frac{9}{2}$

$f(f(6)) = f(\frac{9}{2}) = \frac{9/2 + 3}{9/2 - 4} = 15$

b) to find $f^{-1}(x)$: $f(x) = \frac{x+3}{x-4}$

① write the function using a "y" and set equal to "x" . $x = \frac{y+3}{y-4}$

② rearrange to make y the subject : $xy - 4x = y + 3$

$y = \frac{4x+3}{x-1}$

③ replace y with $f^{-1}(x)$:

$\therefore f^{-1}(x) = \frac{4x+3}{x-1}$

P 7 4 3 2 8 A 0 4 3 2

Question 2 continued

$f^{-1}(x) = \dfrac{4x+3}{x-1}$ $\quad x \in R$, $x \neq 1$

↑ denominator cannot $= 0$ $\therefore x-1 \neq 0$

c) $g(x) = x^2+5$

$g(f(a))$

$= g\left(\dfrac{a+3}{a-4}\right) =$

$= \left(\dfrac{a+3}{a-4}\right)^2 + 5$

$= \dfrac{a^2+6a+9}{a^2-8a+16} + 5 = \dfrac{a^2+6a+9+5a^2-40a+80}{a^2-8a+16}$

$g(f(a)) = 7$

$\therefore \dfrac{6a^2-34a+98}{a^2-8a+16} = 7$

$6a^2-34a+98 = 7a^2-56a+112$

$a^2-22a+14=0$

$a = 11 \cancel{\pm} \sqrt{107}$

reject negative solution as $x>0$ for $g(x)$

3. (a) Using the identity for $\cos(A+B)$, prove that

$$\cos 2A \equiv 2\cos^2 A - 1$$

(2)

(b) Hence, using algebraic integration, find the exact value of

$$\int_{\frac{\pi}{12}}^{\frac{\pi}{8}} (5 - 4\cos^2 3x)\,\mathrm{d}x$$

(4)

3. a) $\boxed{\text{DOUBLE ANGLE FORMULA} \quad \cos(A+B) = \cos(A)\cos(B) - \sin(A)\sin(B)}$

$\cos(2A) = \cos(A+A)$

$= \cos(A)\cos(A) - \sin(A)\sin(A)$

$= \cos^2(A) - \sin^2(A)$ $\leftarrow$ $\boxed{\cos^2(A) + \sin^2(A) = 1}$

$= \cos^2(A) - (1 - \cos^2(A))$

$= 2\cos^2(A) - 1$

b) $\int_{\pi/12}^{\pi/8} (5 - 4\cos^2(3x))\,dx$

$= \int_{\pi/12}^{\pi/8} 5 - 2(2\cos^2(3x))\,dx$ $\leftarrow$ $\boxed{2\cos^2(A) = \cos(2A) + 1}$

$= \int_{\pi/12}^{\pi/8} 5 - 2(\cos(6x) + 1)\,dx$

Question 3 continued

$$= \int_{\pi/12}^{\pi/8} 3 - 2\cos(6x) \; dx$$

$$= \left[3x - \frac{\sin(6x)}{3} \right]_{\pi/12}^{\pi/8}$$

$$= \frac{3\pi}{8} - \frac{1}{3}\sin\left(\frac{6\pi}{8}\right) - \frac{3\pi}{12} + \frac{1}{3}\sin\left(\frac{6\pi}{12}\right)$$

$$= \frac{\pi}{8} + \frac{2-\sqrt{2}}{6}$$

(Total for Question 3 is 6 marks)

4. A new mobile phone is released for sale.

The total sales N of this phone, in **thousands**, is modelled by the equation

$$N = 125 - Ae^{-0.109t} \qquad t \geqslant 0$$

where A is a constant and t is the time in months after the phone was released for sale.

Given that when $t = 0$, $N = 32$

(a) state the value of A.

(1)

Given that when $t = T$ the total sales of the phone was 100 000

(b) find, according to the model, the value of T. Give your answer to 2 decimal places.

(3)

(c) Find, according to the model, the rate of increase in total sales when $t = 7$, giving your answer to 3 significant figures.

(*Solutions relying entirely on calculator technology are not acceptable.*)

(2)

The total sales of the mobile phone is expected to reach 150 000

Using this information,

(d) give a reason why the given equation is not suitable for modelling the total sales of the phone.

(1)

4.a) $N = 125 - Ae^{-0.109t} \qquad t \geqslant 0$

when $t=0$ → $32 = 125 - Ae^{-0.109(0)}$

$32 = 125 - A$

$A = 93$

b) when $t = T$ → $\overset{\text{in thousands}}{100} = 125 - 93e^{-0.109(T)}$

$e^{-0.109T} = \frac{25}{93}$

$-0.109T = \ln\left(\frac{25}{93}\right)$ → $T = 12.05$ months

c) rate of increase $= \frac{dN}{dt}$ when $t=7$

$N = 125 - 93e^{-0.109t}$

$\frac{dN}{dt} = -(93)(-0.109t)\,e^{-0.109t}$

when $t=7$ → $\frac{dN}{dt} = -(93)(-0.109)\,e^{-0.109(7)}$

$= 4.730$ thousand phones per month

$\therefore$ 4730 phones per month

d) $N = 125 - 93e^{-0.109t}$

as $t \to \infty$ $\quad e^{-0.109t} \to 0$

$N = 125 - \cancel{93e^{-0.109t}}^{\,0} \quad \therefore N \to 125$

As the upper limit is 125 000 phones which is below the expected total sales of 150000

5. The curve C has equation

$$y = \frac{\ln(x^2 + k)}{x^2 + k} \qquad x \in \mathbb{R}$$

where k is a positive constant.

(a) Show that

$$\frac{dy}{dx} = \frac{Ax(B - \ln(x^2 + k))}{(x^2 + k)^2}$$

where A and B are constants to be found.

(3)

Given that C has exactly three turning points,

(b) find the x coordinate of each of these points. Give your answer in terms of k where appropriate.

(3)

(c) find the upper limit to the value for k.

(1)

5. a) $y = \dfrac{\ln(x^2+k)}{x^2+k}$ $\quad x \in R$

Quotient rule for differentiating : $y = \dfrac{u}{v} \rightarrow \dfrac{dy}{dx} = \dfrac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

$u = \ln(x^2+k)$ $\qquad \dfrac{du}{dx} = \dfrac{2x}{x^2+k}$

$v = x^2+k$ $\qquad \dfrac{dv}{dx} = 2x$

$$\frac{dy}{dx} = \frac{(x^2+k)\left(\frac{2x}{x^2+k}\right) - (\ln(x^2+k))(2x)}{(x^2+k)^2}$$

$$= \frac{2x(1-\ln(x^2+k))}{(x^2+k)^2}$$

$A = 2$

$B = 1$

Question 5 continued

b) given that C has 3 turning points → at which $\frac{dy}{dx} = 0$

$$\frac{2x(1 - \ln(x^2+k))}{(x^2+k)^2} = 0$$

$$2x(1 - \ln(x^2+k)) = 0$$

$\therefore$ $x = 0$ $\cup$ $\ln(x^2+k) = 1$

$$x^2 + k = e$$

$$x = \pm\sqrt{e-k}$$

c) because there are 3 real coordinates for turning points

$\pm\sqrt{e-k}$ must be real

$\therefore$ $e - k \geq 0$ ← as you can only take a square root of a number ≥ 0

$\therefore$ $k \leq e$

so upper limit of $k = e$

(Total for Question 5 is 7 marks)

6. An area of sea floor is being monitored.

The area of the sea floor, S km^2, covered by coral reefs is modelled by the equation

$$S = pq^t$$

where p and q are constants and t is the number of years after monitoring began.

Given that

$$\log_{10} S = 4.5 - 0.006t$$

(a) find, according to the model, the area of sea floor covered by coral reefs when $t = 2$ **(2)**

(b) find a complete equation for the model in the form

$$S = pq^t$$

giving the value of p and the value of q each to 3 significant figures. **(3)**

(c) With reference to the model, interpret the value of the constant q **(1)**

6. a) $\log_{10}(S) = 4.5 - 0.006t$

when $t=2$ → $\log_{10}(S) = 4.5 - 0.006(2)$

$\log_{10}(S) = 4.488$

$S = 10^{4.488} = 30800 \text{ km}^2$

b) $S = pq^t$

$\log_{10}(S) = \log_{10}(pq^t)$

$\log_{10}(S) = \log_{10}(p) + \log_{10}(q^t)$

$\log_{10}(S) = \log_{10}(p) + t\log_{10}(q)$

$\log_{10}(S) = 4.5 - 0.006t$

(compare to given expression)

LOG RULES

$a\log_b(c) = \log_b(c^a)$

$\log_a b + \log_a c = \log_a(bc)$

$\log_a b = c \rightarrow a^c = b$

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Question 6 continued

$\therefore \quad \log_{10}(p) = 4.5 \qquad \log_{10}(q) = -0.006$

$p = 10^{4.5} = 31600 \qquad q = 10^{-0.006} = 0.986$

$\therefore S = 31600 \times (0.986)^t$

c) The proportion of area covered by coral reefs retained from year to year

as $S = pq^t$

initial area

prop. of area that is kept every year

(Total for Question 6 is 6 marks)

7.

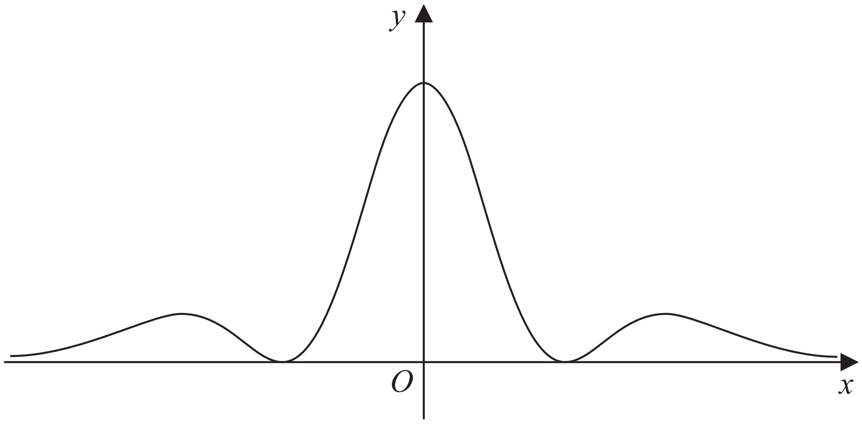


Figure 1

Figure 1 shows a sketch of the curve C with equation $y = \mathrm{f}(x)$ where

$$\mathrm{f}(x) = \mathrm{e}^{-x^2}\left(2x^2 - 3\right)^2$$

(a) Find the range of f

(2)

(b) Show that

$$\mathrm{f}'(x) = 2x\left(2x^2 - 3\right)\mathrm{e}^{-x^2}\left(A - Bx^2\right)$$

where A and B are constants to be found.

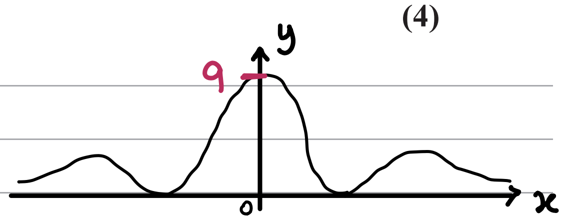
(4)

Given that the line $y = k$, where k is a constant, $k > 0$, intersects the curve at exactly two distinct points,

(c) find the exact range of values of k

(4)

7. a) $f(x) = e^{-x^2}(2x^2-3)^2$



As we can see from the diagram,

the range of $f(x)$ is below the max. point which lies on the y-axis (where $x=0$) & above the x-axis

$f(0) = e^{-(0)^2}(2(0)^2-3)^2 = 9$

$\therefore$ range : $0 \leq f(x) \leq 9$

b) $f(x) = e^{-x^2}(2x^2-3)^2$

$\boxed{\text{PRODUCT RULE : } \quad y = uv \quad y' = u'v + uv'}$

$u = e^{-x^2} \qquad \frac{du}{dx} = -2xe^{-x^2}$

$v = (2x^2-3)^2 \qquad \frac{dv}{dx} = 2 \times (4x) \times (2x^2-3)$

$= 8x(2x^2-3)$

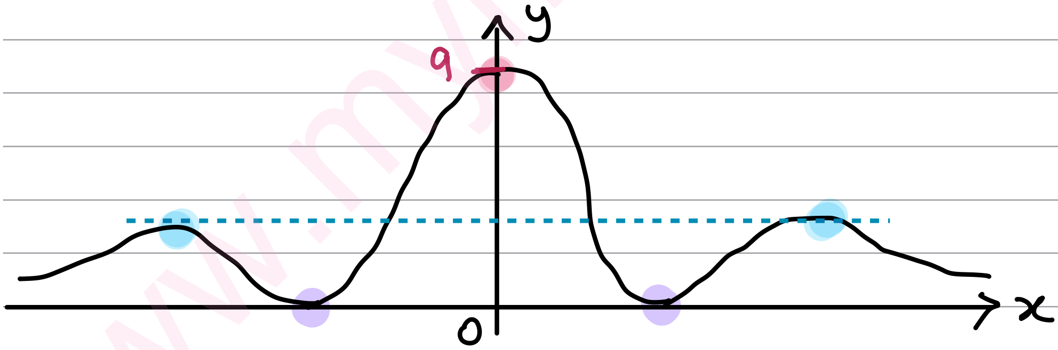
$f'(x) = (-2xe^{-x^2})((2x^2-3)^2) + (e^{-x^2})(8x(2x^2-3))$

$= 2x(2x^2-3)e^{-x^2}(-(2x^2-3)+4)$

$= 2x(2x^2-3)e^{-x^2}(7-2x^2)$

$A = 7$
$B = -2$

c) $y = k$ intersects the curve at exactly 2 distinct points



First we must work out the coordinates of the stationary points :

$f'(x) = 2x(2x^2-3)e^{-x^2}(7-2x^2) = 0 \leftarrow$ (at stationary points, $\frac{dy}{dx} = 0$)

$\hookrightarrow x = 0 \;\cup\; (2x^2-3) = 0 \;\cup\; (7-2x^2) = 0$

Question 7 continued

so stationary points : $x = 0$ $\cup$ $x = \pm\sqrt{\frac{3}{2}}$ $\cup$ $x = \pm\sqrt{\frac{7}{2}}$

$x = 0$: maximum $\rightarrow (0, 9)$

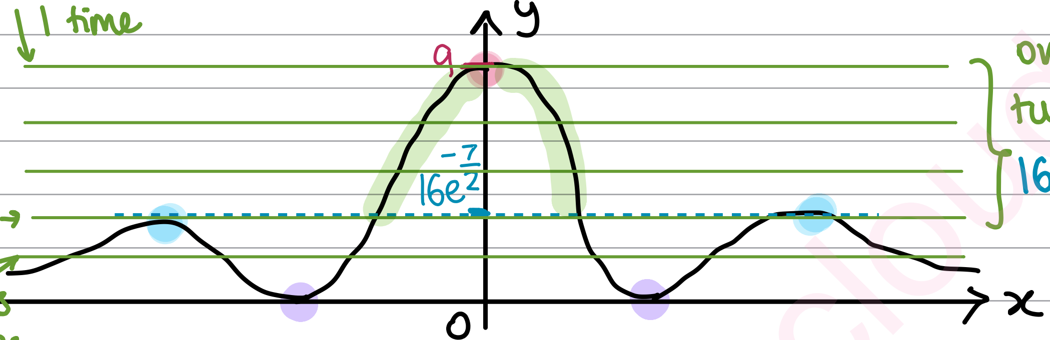
$x = \pm\sqrt{\frac{3}{2}}$: local min $\rightarrow \left(\pm\sqrt{\frac{3}{2}}, 0\right)$

$x = \pm\sqrt{\frac{7}{2}}$: local max $\rightarrow \left(\pm\sqrt{\frac{7}{2}}, 16e^{-\frac{7}{2}}\right)$

Intersects 1 time

Intersects 4 times

intersects 6 times



only intersects twice when $16e^{-\frac{7}{2}} < k < 9$

[or when $k = 0$ but we're given $k > 0$]

$\therefore\ 16e^{-\frac{7}{2}} < k < 9$

8. (a) Prove that

$$2\operatorname{cosec}^2 2\theta(1-\cos 2\theta) \equiv 1+\tan^2\theta$$

(4)

(b) Hence solve for $0 < x < 360°$, where $x \neq (90n)°$, $n \in \mathbb{N}$, the equation

$$2\operatorname{cosec}^2 2x(1-\cos 2x) = 4+3\sec x$$

giving your answers to one decimal place.

(Solutions relying entirely on calculator technology are not acceptable.)

(4)

8. a) LHS : $2\operatorname{cosec}^2(2\theta)\left(1-\cos(2\theta)\right)$ RHS : $1+\tan^2(\theta)$

$$= \frac{2\left(1-\cos(2\theta)\right)}{\sin^2(2\theta)}$$

$\boxed{\cos(2A) = \cos^2(A) - \sin^2(A)}$

$$= \frac{2\left(1-\cos^2(\theta)+\sin^2(\theta)\right)}{\sin^2(2\theta)}$$

$\boxed{\cos^2(A)+\sin^2(A) = 1}$

$$= \frac{2\left(\sin^2(\theta)+\sin^2(\theta)\right)}{\sin^2(2\theta)}$$

$\boxed{\sin(2A) = 2\sin(A)\cos(A)}$

$$= \frac{4\sin^2(\theta)}{\left(2\sin(\theta)\cos(\theta)\right)^2}$$

$$= \frac{\sin^2(\theta)}{\sin^2(\theta)\cos^2(\theta)} = 1 + \frac{\sin^2(\theta)}{\cos^2(\theta)}$$

$$= 1+\tan^2(\theta) = \text{RHS}$$

b) $2\operatorname{cosec}^2(2x)\left(1-\cos(2x)\right) = 4+3\sec(x)$

using identity from (a)

$$1+\tan^2(x) = 4+3\sec(x)$$

$$1+\frac{\sin^2(x)}{\cos^2(x)} = 4+\frac{3}{\cos(x)}$$

multiply through by $\cos^2(x)$

Question 8 continued

$\cos^2(x) + \sin^2(x) = 4\cos^2(x) + 3\cos(x)$

$\cos^2(x) + 1 - \cos^2(x) = 4\cos^2(x) + 3\cos(x)$

$4\cos^2(x) + 3\cos(x) - 1 = 0$

let $\cos(x) = u$

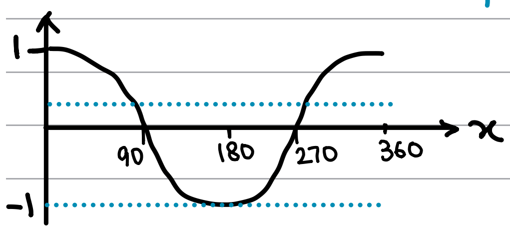
$4u^2 + 3u - 1 = 0$

$(4u - 1)(u + 1)$

$\therefore u = \cos(x) = \frac{1}{4} \cup \cancel{-1}$

$0 < x < 360°$

$x \neq (90n)°$



$\therefore x = 75.5° \cup 284.5°$

$\cup \cancel{180°}$

reject as $180 = 90(2)$

& $x \neq (90n)°$

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9. **In this question you must show all stages of your working.**

Solutions relying on calculator technology are not acceptable.

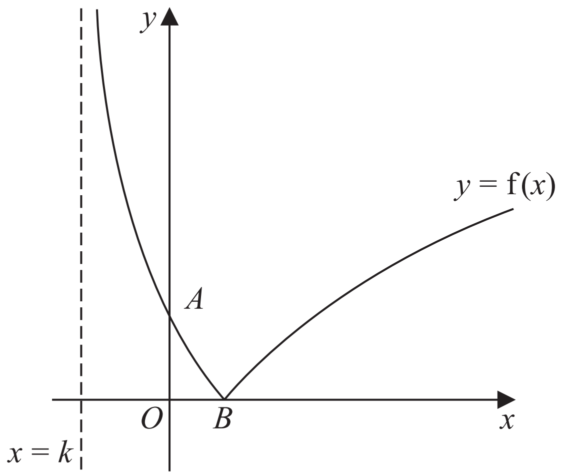


Figure 2

Figure 2 shows a sketch of the curve with equation

$$y = |2 - 4\ln(x + 1)| \qquad x > k$$

where k is a constant.

Given that the curve

- has an asymptote at $x = k$
- cuts the y-axis at point A
- meets the x-axis at point B

as shown in Figure 2,

(a) state the value of k

(1)

(b) (i) find the y coordinate of A

(ii) find the exact x coordinate of B

(3)

(c) Using algebra and showing your working, find the set of values of x such that

$$|2 - 4\ln(x + 1)| > 3$$

(5)

9. a) $y = |2 - 4\ln(x+1)| \qquad x > k$

k is where value for y is undefined → which occurs when $x + 1 = 0$ as $\ln(0)$ is undefined

Question 9 continued

$\therefore \ k = -1$

b)(i) A → y-intercept when $x = 0$

$A = |2 - 4\ln(0+1)| = 2$

(ii) B → x-intercept when $y = 0$

$0 = |2 - 4\ln(B+1)|$

$\ln(B+1) = \frac{1}{2}$

$B + 1 = e^{\frac{1}{2}}$

$B = e^{\frac{1}{2}} - 1$

c) $|2 - 4\ln(x+1)| > 3$

CRITICAL VALUES

$2 - 4\ln(x+1) = 3$

$4\ln(x+1) = -1$

$\ln(x+1) = -\frac{1}{4}$

$x + 1 = e^{-\frac{1}{4}}$

$x = e^{-\frac{1}{4}} - 1$
$(\approx -0.22..)$

$2 - 4\ln(x+1) = -3$

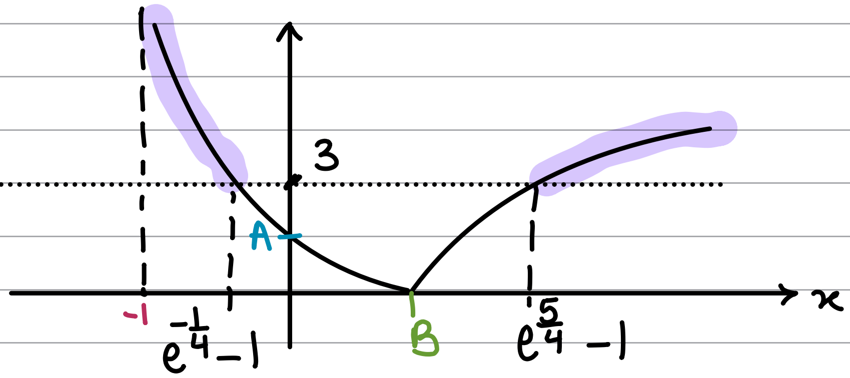
$4\ln(x+1) = 5$

$\ln(x+1) \quad \frac{5}{4}$

$x + 1 = e^{5/4}$

$x = e^{5/4} - 1$
$(\approx 2.49 \ldots)$

Question 9 continued



$\therefore\ f(x) > 3$ when

$-1 < x < e^{-\frac{1}{4}} - 1$

$\cup$

$x > e^{\frac{5}{4}} - 1$

10. **In this question you must show all stages of your working.**

Solutions relying on calculator technology are not acceptable.

A curve C has equation

$$x = \sin^2 4y \qquad 0 \leqslant y \leqslant \frac{\pi}{8} \qquad 0 \leqslant x \leqslant 1$$

The point P with x coordinate $\frac{1}{4}$ lies on C

(a) Find the exact y coordinate of P

(2)

(b) Find $\dfrac{dx}{dy}$

(2)

(c) Hence show that $\dfrac{dy}{dx}$ can be written in the form

$$\frac{dy}{dx} = \frac{1}{\sqrt{q + r(x+s)^2}}$$

where q, r and s are constants to be found.

(3)

Using the answer to part (c),

(d) (i) state the x coordinate of the point where the value of $\dfrac{dy}{dx}$ is a minimum,

(ii) state the value of $\dfrac{dy}{dx}$ at this point.

(2)

10. a) $x = \sin^2(4y) \qquad 0 \leq y \leq \frac{\pi}{8} \qquad 0 \leq x \leq 1$

$P: x = \frac{1}{4} \rightarrow \frac{1}{4} = \sin^2(4y)$

$\frac{1}{2} = \sin(4y) \qquad y = \frac{\pi}{24}$

Question 10 continued

b) $x = \sin^2(4y)$

$$\boxed{\text{CHAIN RULE}: \frac{dy}{dx} = \frac{dy}{\cancel{du}} \times \frac{\cancel{du}}{dx}}$$

$v = \sin(4y)$ $\qquad \frac{dv}{dy} = 4\cos(4y)$

$x = (\sin(4y))^2 = v^2$ $\qquad \frac{dx}{dv} = 2v$

$$\frac{dx}{dy} = \frac{dx}{dv} \times \frac{dv}{dy}$$

$$= 2v \times 4\cos(4y) = 8(\sin(4y))(\cos(4y))$$

c) $$\boxed{\frac{dy}{dx} = \frac{1}{(dx/dy)}}$$ $\qquad \therefore \frac{dy}{dx} = \frac{1}{8(\sin(4y))(\cos(4y))}$

$x = \sin^2(4y)$

$\sin(4y) = \sqrt{x}$

$\cos(4y) = \sqrt{1-\sin^2(4y)} = \sqrt{1-x}$

$$\therefore \frac{dy}{dx} = \frac{1}{8\sqrt{x}\sqrt{1-x}} = \frac{1}{\sqrt{64(x-x^2)}}$$

$$= \frac{1}{\sqrt{-64\left(\left(x-\frac{1}{2}\right)^2-\frac{1}{4}\right)}}$$

$$= \frac{1}{\sqrt{16-64\left(x-\frac{1}{2}\right)^2}}$$

$q = 16$, $r = -64$, $s = -\frac{1}{2}$

Question 10 continued

d) (i) $\frac{dy}{dx}$ is minimum when the denominator has its max value

denominator : $\sqrt{16 \underbrace{- 64\left(x-\frac{1}{2}\right)^2}}$ always neg

In order to maximise the value of the denominator, we must make $-64\left(x-\frac{1}{2}\right)^2 = 0$

$\therefore \quad x = \frac{1}{2}$

(ii) $\frac{dy}{dx}$ when $x = \frac{1}{2} \rightarrow \frac{1}{\sqrt{16}} = \frac{1}{4}$

(Total for Question 10 is 9 marks)

TOTAL FOR PAPER IS 75 MARKS